

1. (10 pts) Find the orthogonal projection of  $u$  on  $v$  where

$$u = \begin{bmatrix} 3 \\ 4 \end{bmatrix}, v = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Ans:

We have  $u \cdot v = -1$  and  $\|v\|^2 = 25$ . So

$$w = \frac{u \cdot v}{\|v\|^2} v = \frac{-1}{25} \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

2. (10 pts) An orthogonal set  $S$  and a vector  $u$  in  $\text{Span } S$  is given. Use dot products to represent  $u$  as a linear combination of vectors in  $S$ .

$$S = \left\{ \begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix} \right\}, u = \begin{bmatrix} 19 \\ 3 \\ 2 \end{bmatrix}.$$

Ans: Let

$$v_1 = \begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix}, v_2 = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}, v_3 = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}.$$

Then  $u \cdot v_1 = -14$ ,  $u \cdot v_2 = -12$ ,  $u \cdot v_3 = 84$ . Also  $\|v_1\|^2 = 14$ ,  $\|v_2\|^2 = 6$ ,  $\|v_3\|^2 = 21$ . Therefore,  $u = c_1 v_1 + c_2 v_2 + c_3 v_3$  where

$$\begin{aligned} c_1 &= \frac{u \cdot v_1}{\|v_1\|^2} = -1 \\ c_2 &= \frac{u \cdot v_2}{\|v_2\|^2} = -2 \\ c_3 &= \frac{u \cdot v_3}{\|v_3\|^2} = 4 \end{aligned}$$